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A Robust State Estimator Based on Maximum Exponential Square (MES)

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Introduction

- How to suppress bad data in SE?
  - Largest normalized residual (LNR) approach
    - Residual smearing problem ?
  - Robust estimator
    - M-estimators (Such as WLAV, QC, QL, SHGM)
    - leverage bad data ?
    - Calculation speed ?

Introduction(2)

- The Proposed Maximum Exponential Square (MES) Estimator
  - Differentiable objective function
  - Avoid leverage point problem
  - Strong ability to suppress bad data
  - Similar implementation with WLS estimator
  - Fast calculation speed (approaches the speed of FDSE+LNR)

Maximum Exponential Square (MES) Model

For measurement equation

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{e}$$

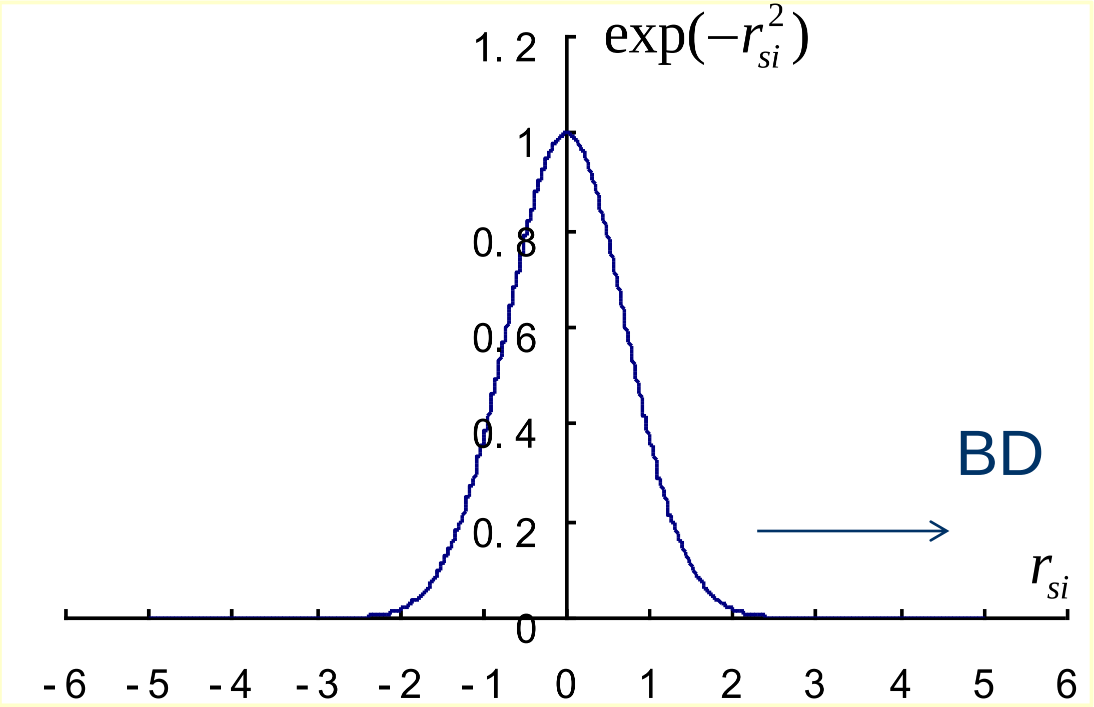
A maximization problem with exponential square objective function

$$\max_{\mathbf{x}} \quad J(\mathbf{x}) = \sum_{i=1}^{m} w_i \exp(-r_{si}^2)$$

$$r_{si} = \frac{z_i - h_i(\mathbf{x})}{\sqrt{2}\sigma}$$

Mathematical Characteristics

- Larger residual has less impact on objective function



Explained by information theory

- Parzen window method with Gaussian kernel:

The estimated pdf for random variable γ

$$f_e(\boldsymbol{\gamma} = \mathbf{x}) = \frac{1}{m\sigma^n}\sum_{i=1}^{m}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(\mathbf{x}-\mathbf{z}_i)^T(\mathbf{x}-\mathbf{z}_i)}{2\sigma^2}\right) \qquad (1)$$

  - σ: The width of Parzen window
  - A nonparametric estimation method
  - No prior knowledge about the random variables' distribution type is needed, so gross errors can be treated in Parzen window method

Explained by information theory(2)

- If the information loss of the estimator is measured by Renyi's quadratic entropy as follows

$$H_2 \;\; \mathbf{e} \;\; = -\log \int_{-\infty}^{\infty} f_e^2 \;\; \mathbf{e} \;\; d\mathbf{e} \qquad\qquad 2$$

- Maximizing (1), as done in MES method, is just to minimize (2), the information loss of the estimator

MES vs WLS

- If no gross error exists, the MES estimator can be expanded into a Taylor series such as

$$\max_{\mathbf{x}} J(\mathbf{x}) = \sum_{i=1}^{m} w_i \exp\left(-r_{si}^2\right) = \sum_{i=1}^{m} w_i \left(1 - r_{si}^2 + o\left(r_{si}^4\right)\right)$$

i. e. …………MES

$$\min_{\mathbf{x}} \sum_{i=1}^{m} w_i r_{si}^2$$

…………………WLS

Which is similar with WLS estimator

Solution Method

-Optimization condition:

$$\frac{\partial J}{\partial \mathbf{x}} = \sum_{i=1}^{m} \frac{\partial \omega_i}{\partial \mathbf{x}} = \mathbf{H}^T \mathbf{W}(\mathbf{x})(\mathbf{z} - \mathbf{h}(\mathbf{x})) = \mathbf{0} \quad \Rightarrow \quad f(x) = 0$$

$$\omega_i(\mathbf{x}) \square \; w_i \exp(-r_{si}^2) \qquad W_{ii}(\mathbf{x}) \square \; \omega_i(\mathbf{x}) / \sigma^2$$

–Hessian matrix:

$$\frac{\partial^2 J}{\partial \mathbf{x}^2} = -\mathbf{H}^T \mathbf{W}(\mathbf{x}) \left[\mathbf{I} - diag\{ \frac{\mathbf{z} - \mathbf{h}(\mathbf{x})}{\sigma^2}^{2} \} \right] \mathbf{H} @ \mathbf{Q} \quad \Rightarrow \quad \frac{\partial f}{\partial x}$$

–Newton Iteration:

$$\mathbf{Q}\Delta\mathbf{x}^k = \text{-}\mathbf{H}^T \mathbf{W}(\mathbf{x}^k)(\mathbf{z} - \mathbf{h}(\mathbf{x}^k))$$

$$\mathbf{x}^{k+1} = \mathbf{x}^k + \Delta\mathbf{x}^k$$

$$\frac{\partial f}{\partial x} \Delta x = -f(x)$$

Leverage Point Problem

- The relationship between residual *r* and measurement error *e*

$$\mathbf{r} = \mathbf{K}\mathbf{e}$$

  - **K**: residual sensitivity matrix
- For WLS estimator:

$$\mathbf{K}^{WLS} = \mathbf{I} - \mathbf{H}\left(\mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^{T}\mathbf{R}^{-1}$$

  - The K^{WLS} is less relevant with measurement values
  - $K_{ii} \approx 0$ for a leverage point

Leverage Point Problem(2)

- For MES estimator:

$$\mathbf{K}^{MES} = \mathbf{I} - \mathbf{H}\mathbf{Q}^{-1}\mathbf{H}^{\mathbf{T}}\mathbf{W}$$

  - The K^{WLS} is relevant with measurement value
  - For the bad measurement i with a large residual,

$$W_{ii}(\mathbf{x}) \square \omega_i(\mathbf{x}) / \sigma^2 \approx 0 \quad \omega_i(\mathbf{x}) \square w_i \exp(-r_{si}^2)$$

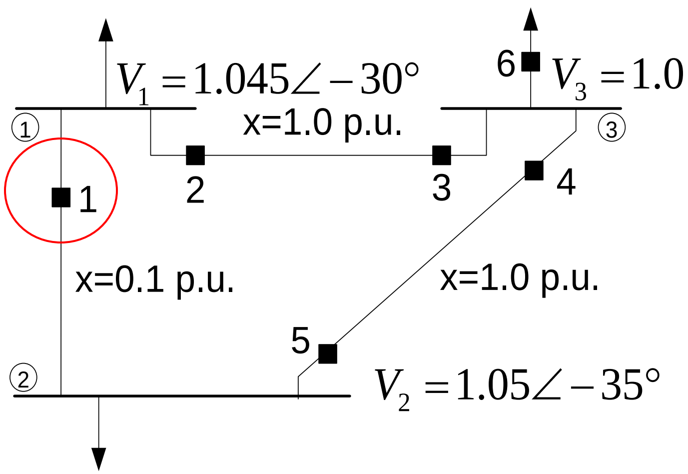
  - the corresponding

$$K_{ii}^{MES} \approx 1$$

  - MES estimator can successfully identify BD even at conventional leverage point.

Leverage Point Test

– Measurement 1 is a leverage point in WLS estimator (due to the small reactance of branch 1-2)
– Gross error is added to measurement 1
– MES estimator did not suffer leverage point problem.



The values of K_{11}^{WLS} and K_{11}^{MES} in 3-bus system

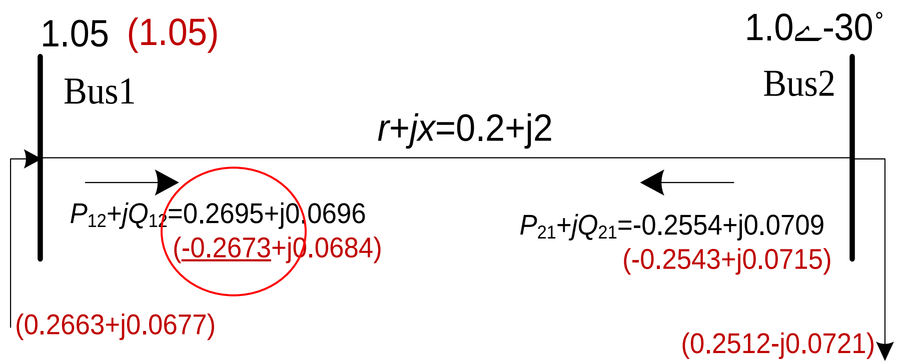
Type	K_{11}^{WLS}	K_{11}^{MES}
P	0.0091	1.0000
Q	0.0098	1.0000

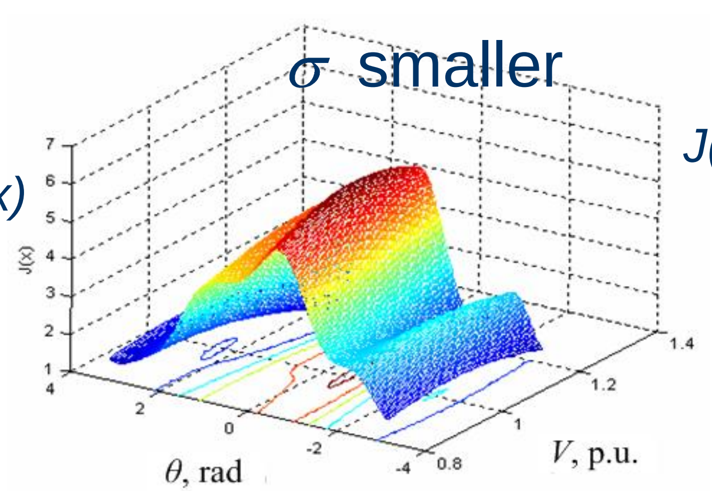
Global Optimal Point

- MES method is similar to Parzen window method.
- In the Parzen window method, the average value of pdf in the Parzen window is assumed as the value of pdf at the window's centre point.
- If σ is smaller, the assumption is correct.
- If σ is larger, optimal solution may deviate from true value.
- σ should be adjusted from larger to smaller so as for the MES estimator to reach the true optimal solution

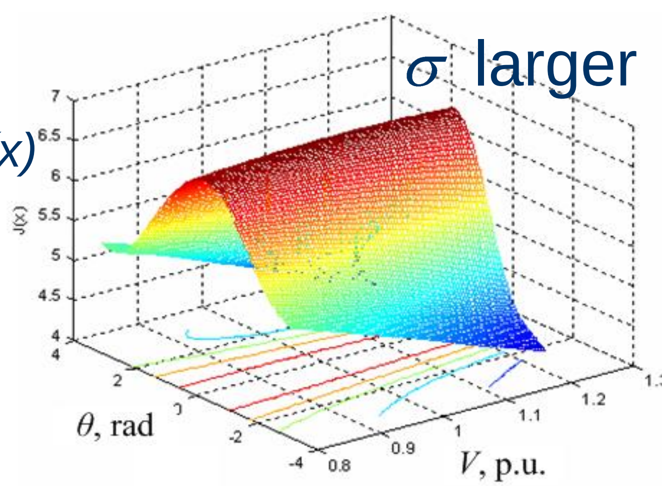
Global Optimal Point(2)

One BD





(a) $2\sigma^2 = 0.1$



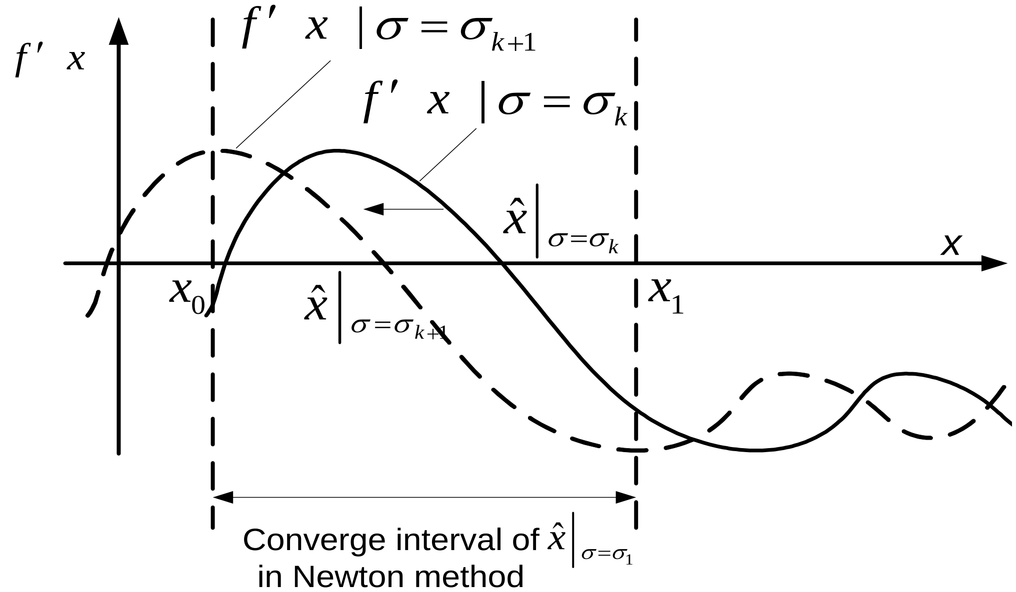
(b) $2\sigma^2 = 1$

J(x) in 2-bus system

How to get global optimal point

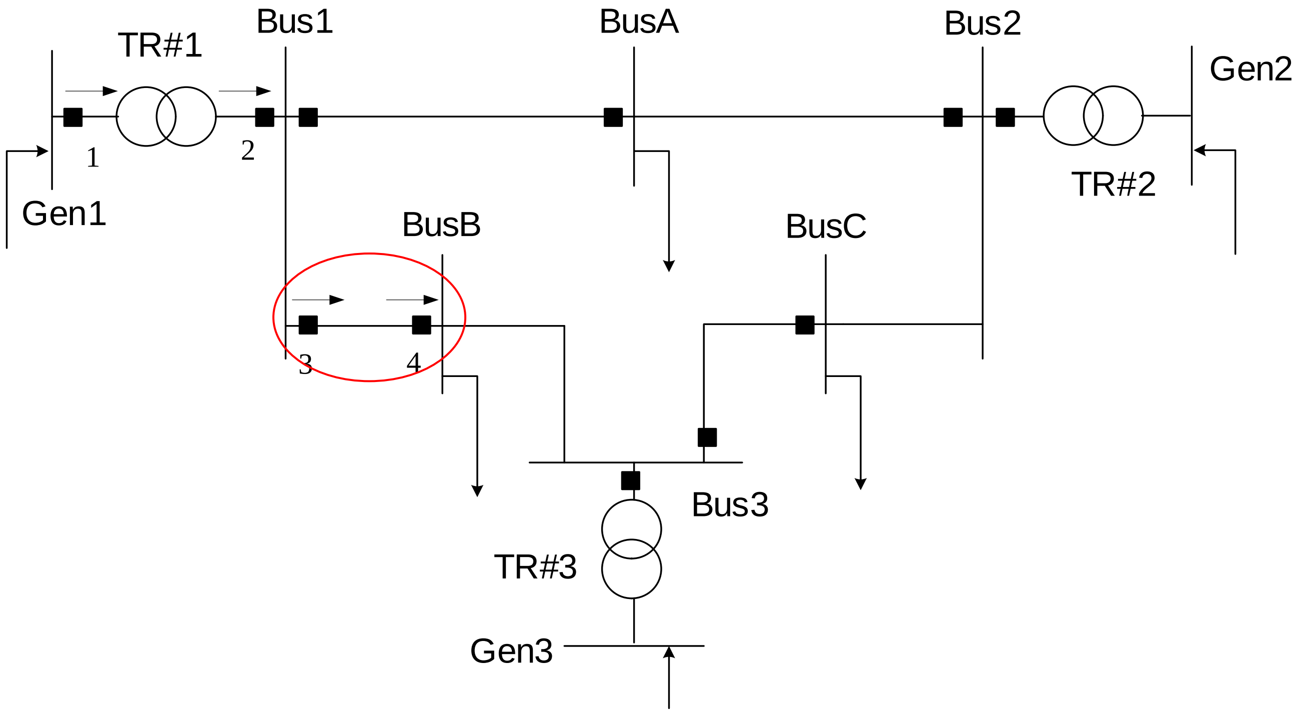
- adjust σ step by step from a larger to a smaller.
- The converge domain of Newton method:
  - Keep Hessian matrix $\boldsymbol{f}'(\boldsymbol{x})$ negative definite at each adjust step

$$\boldsymbol{f}(\boldsymbol{x}) = \frac{\partial J(\boldsymbol{x})}{\partial \boldsymbol{x}} = \boldsymbol{0}$$



Numerical Results

- 9-bus system
  - Conforming errors at 3rd and 4th



Numerical Results(2)

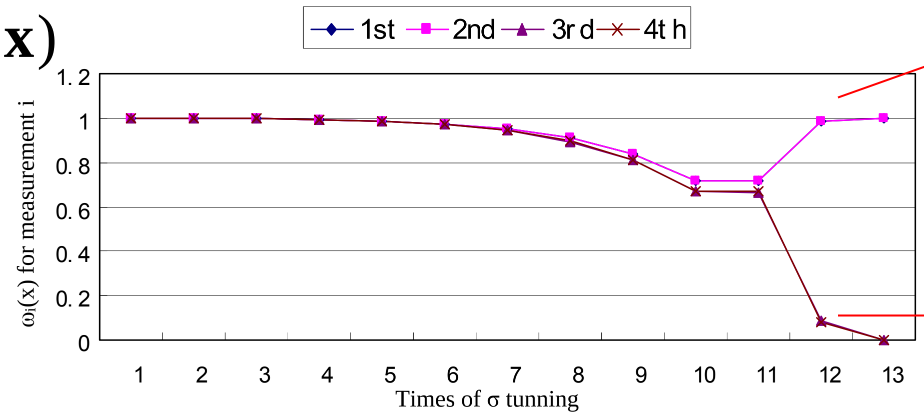
Bad Data — wrongly estimated — Correctly estimated

Measurement Number	True value ①	Meas. value ②	FDSE+LNR method		MES method		
			Estimated Meas. ③	Meas. est. Error ①-③	Estimated Meas. ④	Meas. est. error ①-④	$\omega(\mathbf{x})$
1 st	71.64	71.64	51.61	20.03	71.71	-0.07	1.0000
2 nd	-71.64	-71.64	-51.61	-20.03	-71.71	0.07	1.0000
3 rd	30.70	**10.70**	11.32	19.38	30.82	-0.12	8.207×10^{-6}
4 th	-30.57	**-10.57**	-11.41	-20.00	-30.53	-0.04	8.951×10^{-6}

- LNR method fails to identify these 2 conforming errors
- MES estimator estimates accurate results directly

Numerical Results(3)

$\omega(\mathbf{x})$

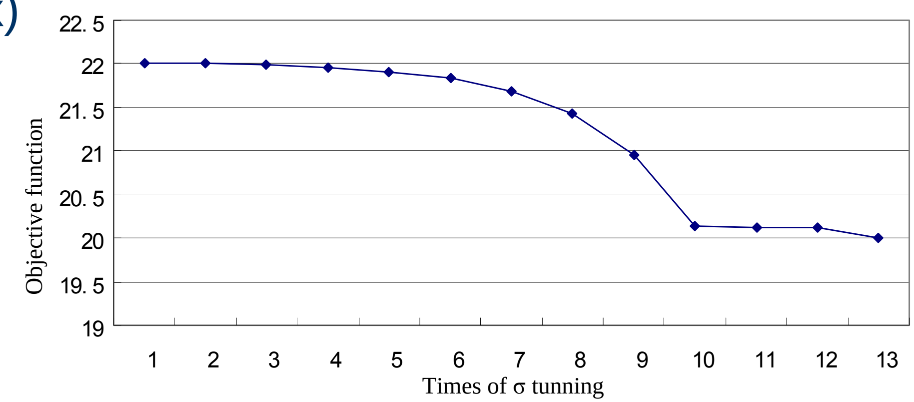


Good data

$$\omega_i(\mathbf{x}) \square\ w_i \exp(-r_{si}^2)$$

Bad data

$J(\mathbf{x})$



$$J(\mathbf{x}) = \sum_{i=1}^{m} \omega_i(\mathbf{x})$$

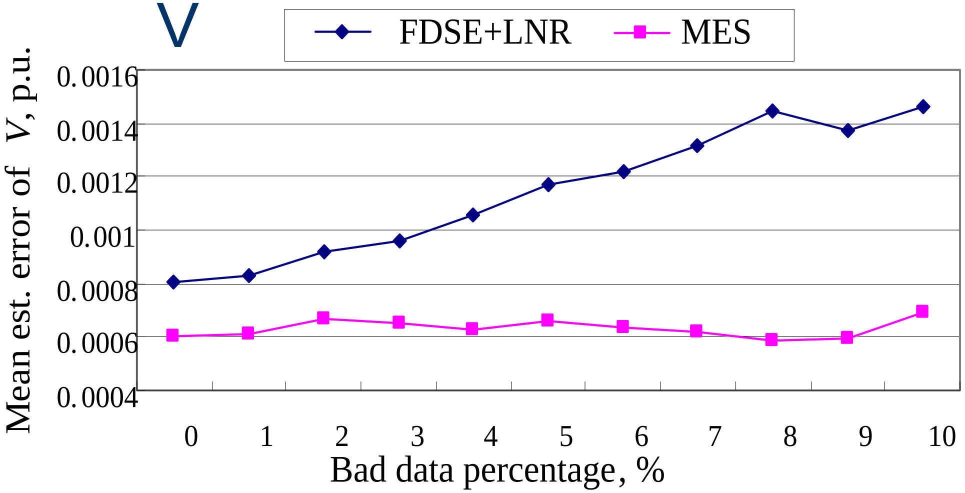
From 22 down to 20

Numerical tests

- 118-bus system
  - Totally 11 different bad data percentages ranging from 0% to 10% are generated
  - For each bad data percentage, 30 different cases are randomly produced and calculated
  - the averages of mean absolute estimation errors of bus voltages and angles under each bad data percentage are recorded

Numerical tests(2) (Average est. err.)

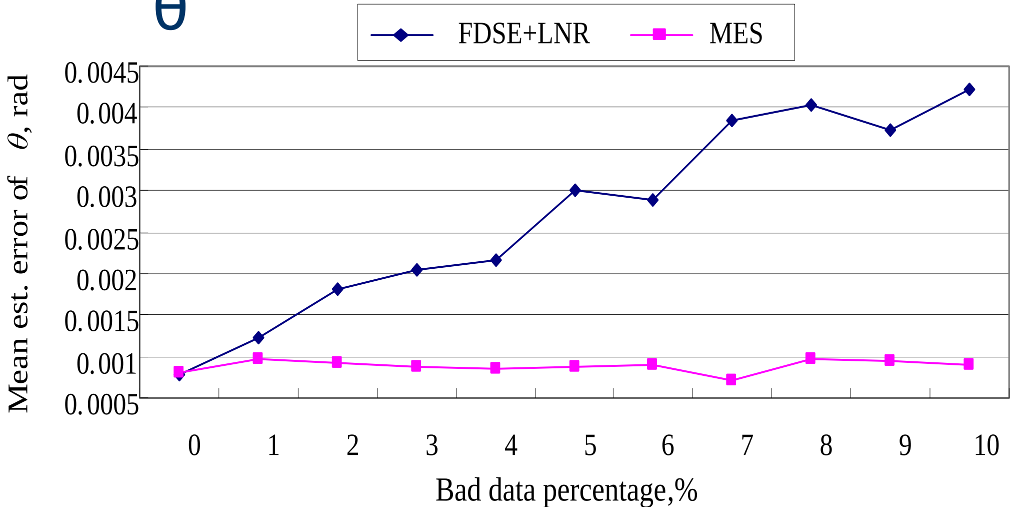
V



FDSE+LNR

MES

θ



FDSE+LNR

MES

Practical Applications

- A provincial power system in Central-China

546 buses, 737 branches, daily peak load 8GW

- Measurement acceptance rate index :

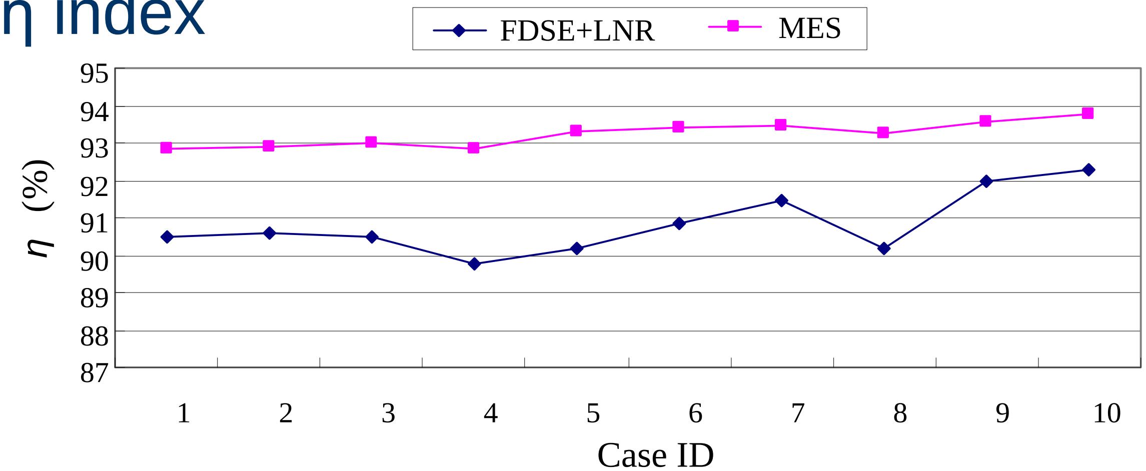
$$\eta = \frac{n_\xi}{m} \times 100\%$$

n_ξ : the number of measurements whose residual is less than a threshold ξ.

m : the number of whole measurements

Practical Application(2)

η index

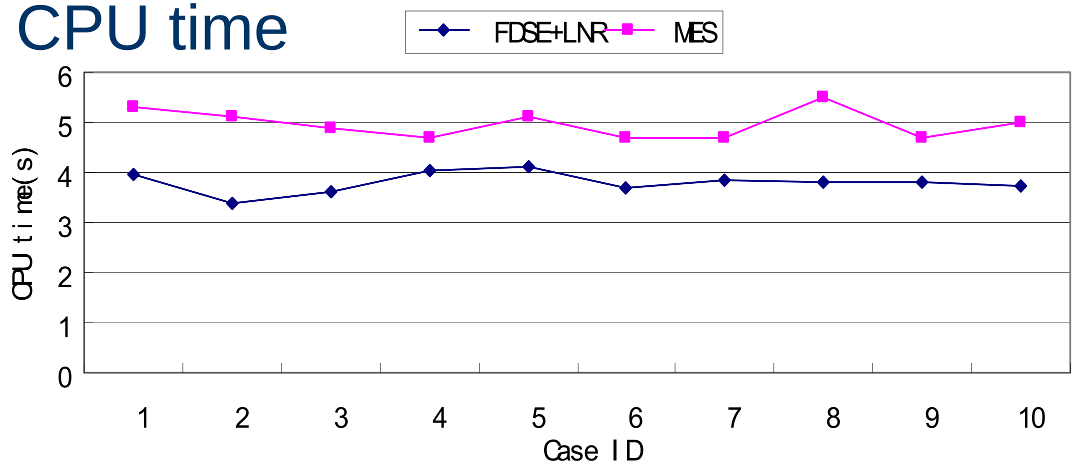


MES(93-94%)

FDSE+LNR

(90-92%)

CPU time

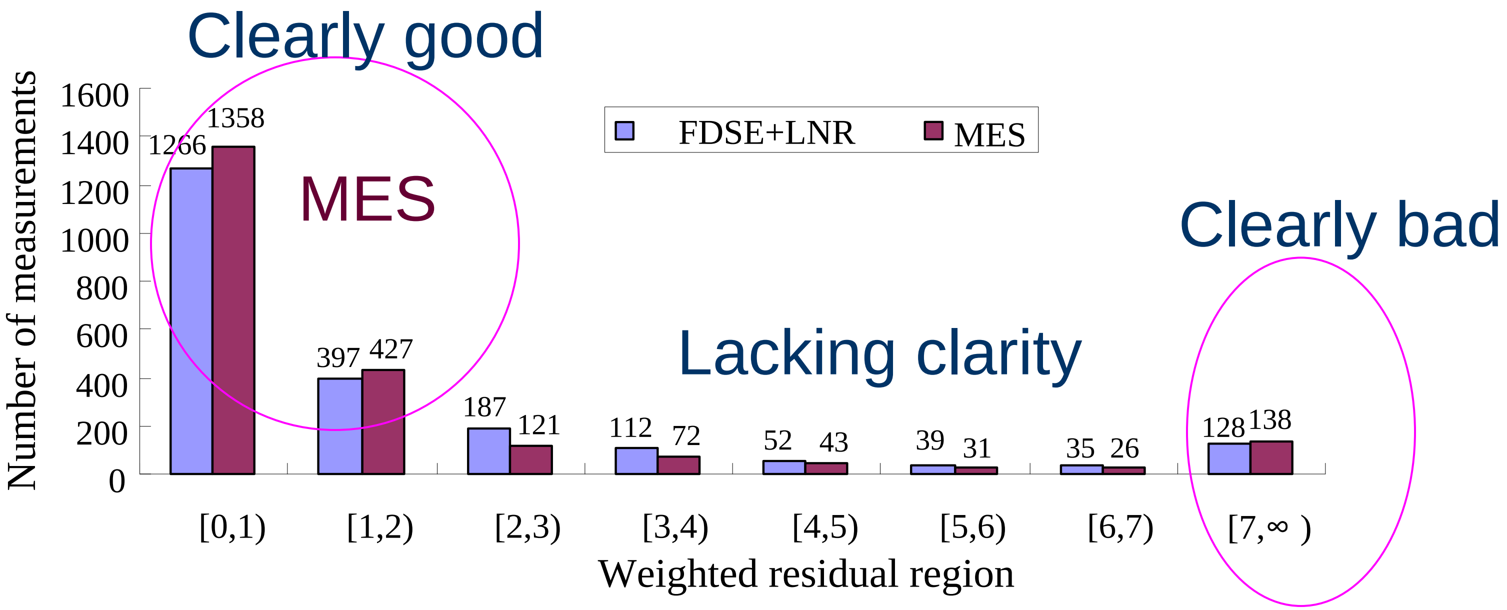


MES(5 sec)

FDSE+LNR(4 sec)

Practical Application(3)

- Residual distribution:
  - MES estimator separates more residuals to a small region and to a significant large region.



Thank You!