

# Believability Evaluation of a State Estimation Result

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# Content

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- **Introduction**
- **Problems in SE Application**
- **Conventional Indices to Evaluate Believability of SE Results**
- **A Correntropy(COE) Based Index**
- **Numerical Test Results**
- **Conclusion**

# Introduction

- **Problems in State Estimation Applications**
  - Accuracy deteriorated by bad data
  - Zero injection constraints
- **How to Evaluate the Believability of a SE Result?**
  - Lab: Compare estimation result with the simulated true power flow
  - Real systems: true state is unknown, how to do?
- **The only known is measurement, How to evaluate SE believability only by measurement**

# Existing Problem (1) – Bad Data

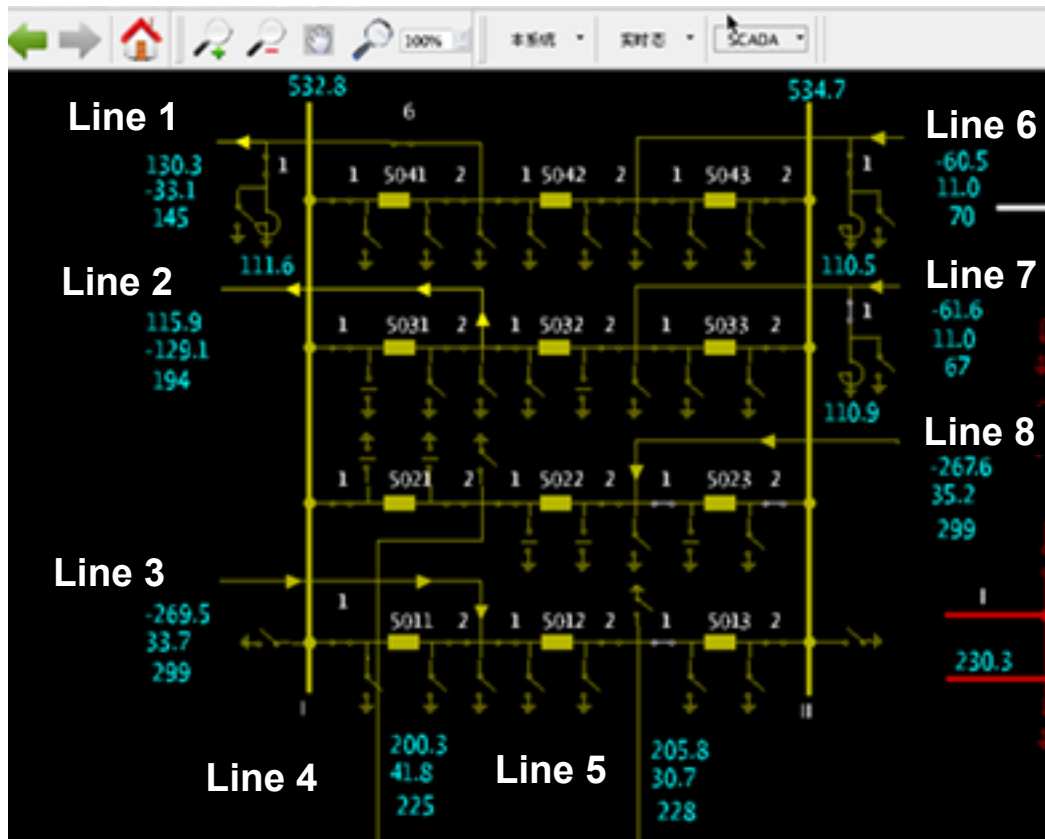
- **Weighted Least Square (WLS)**
  - Assumption: measurement errors accord with Gaussian distribution. not true in real power systems, (GD, BD)
  - Largest Normalized Residual identification, fail especially for conforming BD
- **Robust SE: WLAV, QC, QL, SHGM (Mili L., 2006), etc**
  - Inefficient
  - Empirical
  - Not global differentiable
  - Difficult to identify bad data on leverage points

# Existing Problem (2) – Zero Injection Constraint

- **Large Weight Strategy in WLS:**
  - Too large a weight → Numerical problem
  - Too small a weight → KCL not satisfied
- **Problem in Practical Application**
  - A SE result seems OK , but the following Load Flow will stray from SE
  - Following EMS application software can not work normally.
  - It is very common in China EPCC

# Existing problem (3) -- Example

A 500 kV ST Example



- SCADA results
- Q Meas. (Mvar)

Line 1: -33.1

Line 2: -125.1

Line 3: 33.7

Line 4: 41.8

Line 5: 30.7

Line 6: 11.0

Line 7: 11.0

Line 8: 35.2

SCADA

Line 1  
P: 130.3, Q: -33.1

Line 5  
P: 205.8, Q: 30.7

Line 2  
P: 115.9, Q: -125.1

Line 6  
P: -60.5, Q: 11.0

Line 3  
P: -269.5, Q: 33.7

Line 7  
P: -61.6, Q: 11.0

Line 4  
P: 200.3, Q: 41.8

Line 8  
P: 267.6, Q: 35.2

500kV bus

- $\sum \text{SCADA} = 5.2 \text{ Mvar (3.3\%)}$
- Measurements are good through manually checked

# Existing problem (4)

- SE results
- Q Esti./Meas. (MVar)
  - Line 1: -53.5/-33.1
  - Line 2: -164.3/-125.1
  - Line 3: 34.5/33.7
  - Line 4: -5.9/41.8
  - Line 5: -20.4/30.7
  - Line 6: 89.0/11.0
  - Line 7: 88.7/11.0
  - Line 8: 35.9/35.2

## State Estimation

Line 1 P: -0.6, Q: -53.5	Line 5 P: 130.8, Q: -20.4
Line 2 P: -3.2, Q: -164.3	Line 6 P: 24.4, Q: 89.0
Line 3 P: -153.8, Q: 34.5	Line 7 P: 24.6, Q: 88.7
Line 4 P: 131.9, Q: -5.9	Line 8 P: -154.3, Q: 35.9

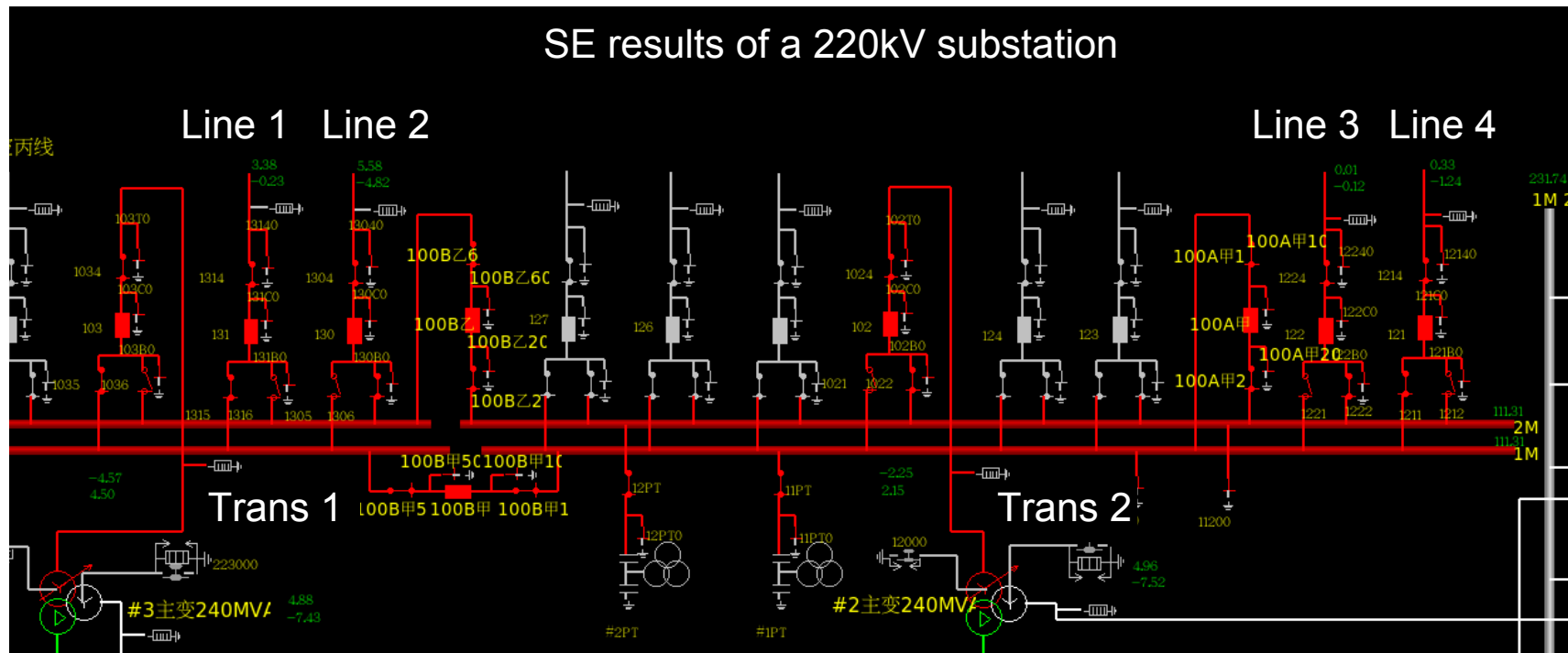
500kV bus

- $\sum SE = 4.0 \text{ MVar} \neq 0$
- SE results are terribly bad through manually checked



# Existing problem (5)

- 220 kV ST Example (another system)



# Existing problem (6)

- 220 kV ST Example (to check zero injections constraint)

- Estimated P Meas. (MW)

- Line 1: 3.38
- Line 2: 5.58
- Line 3: 0.01
- Line 4: 0.33
- Trans 1: -4.57
- Trans 2: -2.25

- $\sum = 2.48\text{MW} \neq 0$
- SE results are bad

State Estimation

Line 1

P: 3.38, Q: -0.23

Line 2

P: 5.58, Q: -4.82

Line 3

P: 0.01, Q: -0.12

Line 4

P: 0.33, Q: -1.24

Trans 1

P: -4.57, Q: 4.50

Trans 2

P: -2.25, Q: 2.15

220kV bus

# Solution Method

- **For bad data identification problem**, a new robust estimator based on maximum correntropy criterion is developed

[1]Wu W.C., Guo Y., Zhang B.M., and Sun H.B., Robust state estimation method based on maximum exponential square (MES), *IET Proc. on GTD*, vol. 5, no. 11, pp1165-1172, 2011

- **for zero injection constraints problem**, an efficient approach considering rigorous zero injection constraints is proposed

[1]Ye Guo, Boming Zhang, etc, An Efficient State Estimation Algorithm Considering Zero Injection Constraints, *IEEE Trans. on PWRS*, vol. 28, no.3, pp. 2651-2659, 2013

# Conventional Indices to Evaluate Believability of SE Results

- **No an international standard** until recently
  - Common indices: MAR (Mean of absolute values of residuals), MSR (Mean of square of residuals)
  - The WLS objective value (MSR) is not a good index

- **In China: Acceptance rate**

$$\eta = \frac{n_{\xi}}{m} \times 100\%$$

- $n_{\xi}$ : the number of measurements whose residuals less than given empirical thresholds (e.g. 500kV power measurements: 21.6MW)
- $m$ : the total number of measurements
- Plausibility: Yes or No, good or bad measurements
- Drawbacks: 0-1 manner, empirical thresholds, how much good, how much bad?

# Correntropy(COE) based Index (1)

- **Correntropy**: Measure for similarity of two random variables  $\mathbf{z}$  and  $\mathbf{h}(\mathbf{x})$  in information theory
  - Equivalent to the quadratic Renyi's entropy between  $\mathbf{z}$  and  $\mathbf{h}(\mathbf{x})$
  - Non-parametric estimation, no assumption on error model
- **Map point  $\mathbf{z}$  and point  $\mathbf{h}(\mathbf{x})$  from measurement space to a reproduced kernel Hilbert space with kernel function  $K$**
- Use inner product to evaluate the correlation

$$\langle \varphi(\mathbf{z}), \varphi(\mathbf{h}(\mathbf{x})) \rangle = K(\mathbf{z}, \mathbf{h}(\mathbf{x}))$$

- $K$  is referred to as kernel function, usually use Gauss kernel

$$K(\mathbf{z}, \mathbf{h}(\mathbf{x})) = \frac{1}{\sqrt{2\pi}\sigma} \sum_{i=1}^m \exp\left(-\frac{(z_i - h_i(\mathbf{x}))^2}{2\sigma^2}\right)$$

## Correntropy based Index(2)

$$\xi = \frac{1}{m} \sum_{i=1}^m \exp \left[ -\frac{(z_i - h_i(\hat{\mathbf{x}}))^2}{2\delta_i^2} \right] \times 100\%$$

	Acceptance rate (AR)	Correntropy based index(COE)
Theoretical basis	Empirical	Similarity measure in information theory
Measurement partition	0-1, either good or bad	Score each meas. continuously according to its residual
Thresholds	Empirical	not needed

## Correntropy based Index(3)

- **Zero injection constraints must be satisfied strictly**
- **Count mean MW/Mvar mismatches for 0-injection bus**

$$\Delta = \frac{1}{n_Z} \left( \sum_{i \in Z} |P_i| + \sum_{i \in Z} |Q_i| \right)$$

- **Z**: zero injection bus set
- $n_Z$ : number of zero injection buses
- **If Large  $\Delta$**  : SE results do not yield to KCL, erroneous state estimate
- **High COE index & small  $\Delta$**  = believable SE result
- **Low COE index | large  $\Delta$**  = unbelievable SE result

# Numerical tests: 9-bus test system

- Three SE results produced by different estimators
  - **A**: WLS estimator with large weights for zero injections.
  - **B**: WLS estimator with small weights for zero injections (significant power mismatches)
  - **C**: Robust estimator considering rigorous zero injection constraints

Indices	A	B	C
Acceptance rate (%)	100%	100%	91.67%
COE Index (%)	83.74%	90.65%	91.15%
Mean power mismatches of zero injection buses (p.u.)	0.0053	0.0445	7.13e-5
Mean estimation error (p.u.)	0.0047	0.0042	7.44e-4

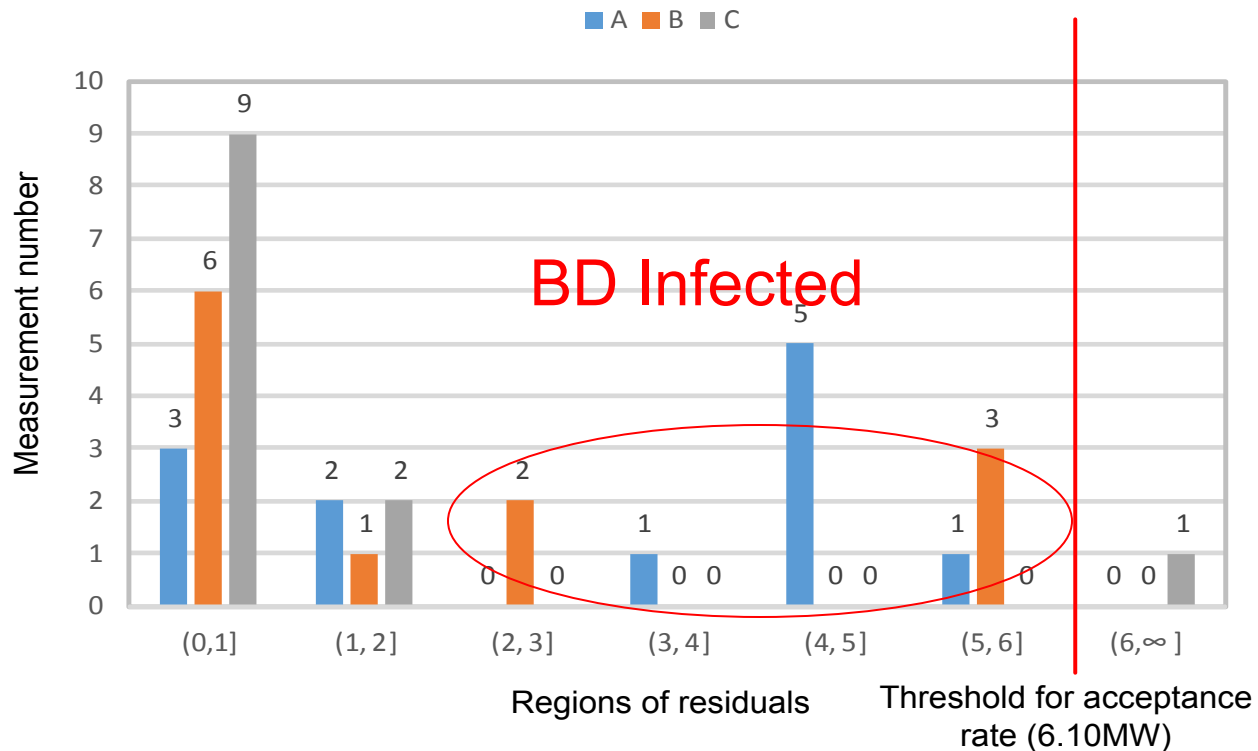
- Acceptance rate gives erroneous conclusions on SE accuracy
- Zero injection examination is necessary



# Numerical tests: 9-bus test system

- **Distribution of residuals**

- A & B: Several meas. reside in the middle residual regions
- C: One identified as bad data, all others have small residuals



# Numerical tests: 118-bus test system

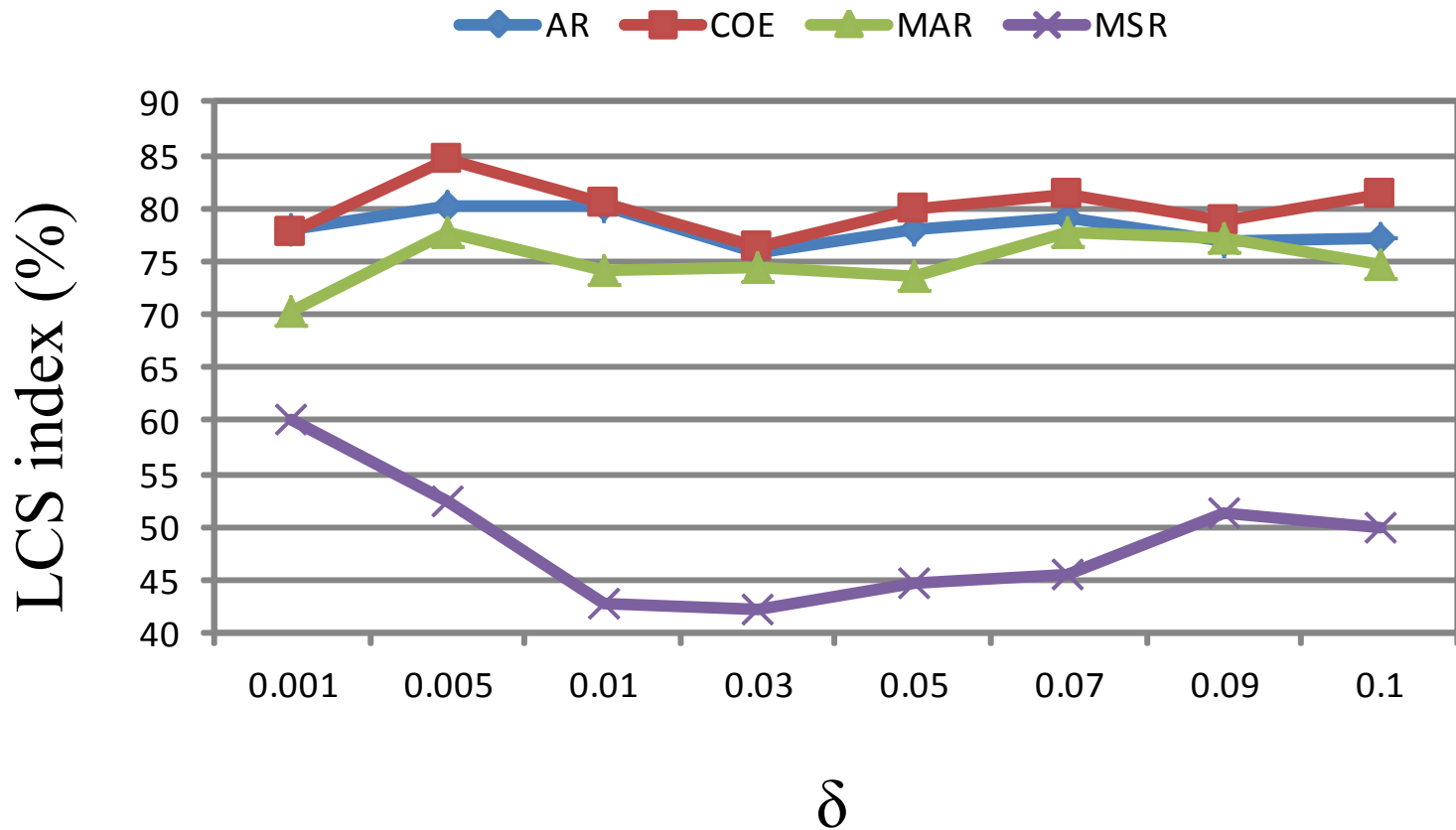
- **Comprehensive comparison**

- Four indices: **Acceptance rate (AR)**, **Correntropy (COE)**, **Mean Absolute value of Residuals (MAR)**, **Mean Square of Residuals (MSR)**
- Change measurement errors, for each point of measurement error, 100 different measurement sets are generated.
- For each measurement set, 10 different SE results are obtained by different estimators
- Count the mean values of the longest common sequence (LCS)

$$LCS \text{ index} = l_{LCS} / 10 \times 100\%$$

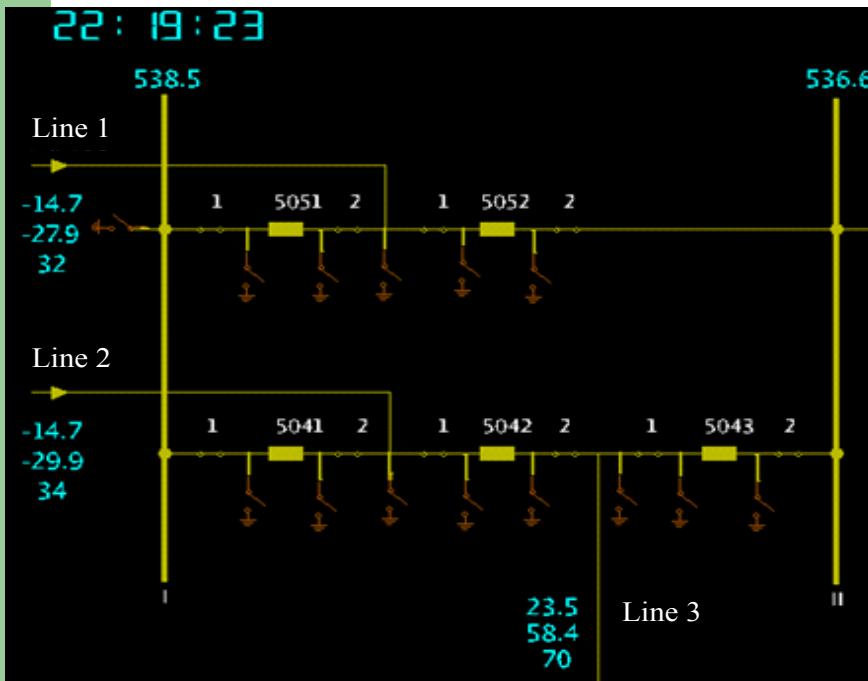
# Numerical tests: 118-bus test system

COE > AR > MAR > MSR

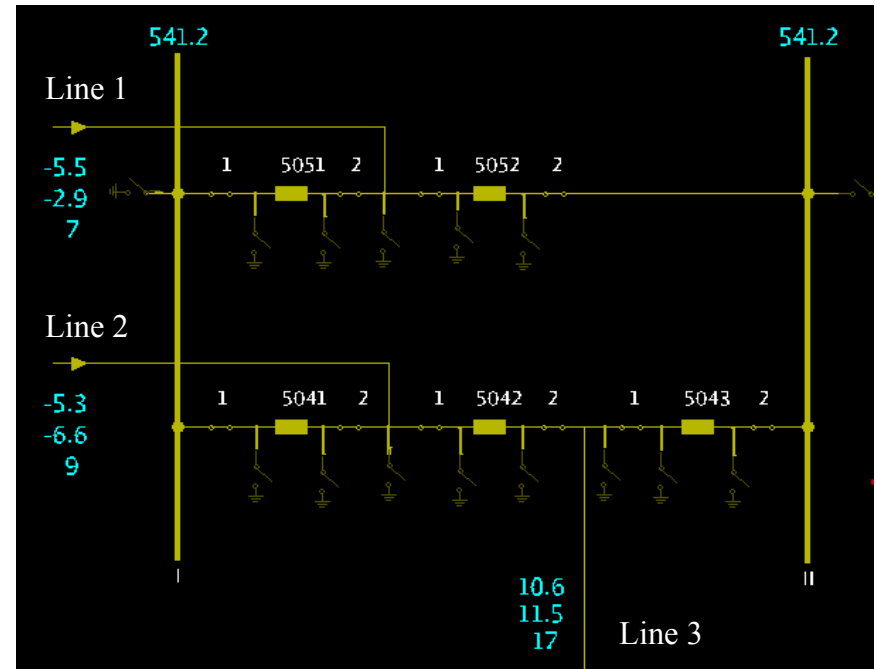


# Numerical tests: a practical system

- A selected 500kV ST - Detail



SCADA



State Estimation

# Numerical tests: a practical system

Q Meas./**Estimated** (MVar)

Line 1: -27.9/**-2.9**

Line 2: -29.9/**-6.6**

Line 3: 58.4/**11.5**

$\sum$ SCADA = 0.6MVar (good)

$\sum$ **SE** = 2.0MVar (bad)

- SE result is not believable
- KCL is not satisfied

SCADA

Line 1

P: -14.7, Q: -27.9

Line 2

P: -14.7, Q: -29.9

500kV bus

Line 3

P: 23.5, Q: 58.4

State Estimation

Line 1

P: -5.5, Q: -2.9

Line 2

P: -5.3, Q: -6.6

500kV bus

Line 3

P: 10.6, Q: 11.5

# Numerical tests: a practical system

- A provincial system in China

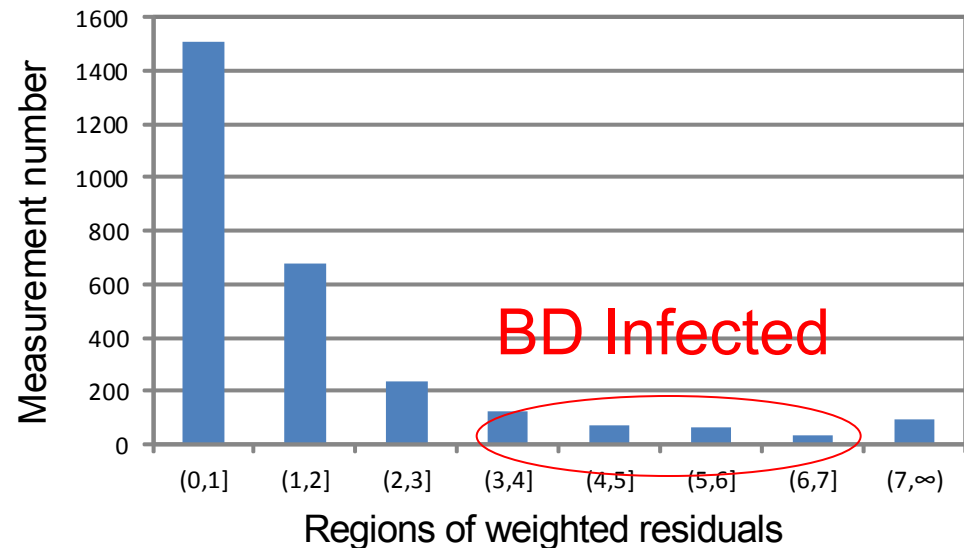
- Some indices

- High acceptance rate
- Low COE Index
- High zero injection mismatch

Acceptance Rate	99%	High
COE Index	77.40%	Low
Mismatches of zero injections	2.08MV A	Large

- Distribution of weighted residuals

- Many measurements are infected!
- Inaccurate SE results!



# Conclusions

- Problems in SE application
    - Bad data
    - Zero injection
  - How to assess the Believability of a SE result
    - Conventional: MAR, MSR, Acceptance rate are not reliable
    - Correntropy (COE) based index
      - + Zero injection constraints checking
- Is a reasonable solution.



**Thank You!**